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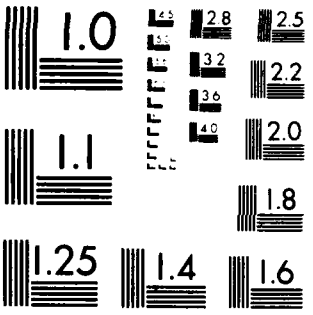

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On Acyclic Database Decompositions

by

Catriel Beeri and Moshe Vardi

Department of Computer Science

Stanford University
Stanford, CA 94305

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Given a universal relation scheme, presented as a set of attributes and a set of dependencies, it may be advantageous to decompose it into a collection of schemes, each with its own sets of attributes and dependencies, which has some desired properties. A basic requirement for such a decomposition to be useful is that the corresponding decomposition map on universal relations be injective. A central problem in database theory is to find the reconstruction map, i.e., the inverse map of an injective decomposition map. The authors prove here that when the decomposition, viewed as a hypergraph, is acyclic and (CONTINUED)

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ITEM #20, CONTINUED: the given dependencies are full implicational dependencies, then the reconstruction map is the natural join. Based on this, the authors show that there is a polynomial time algorithm to test for injectiveness of decompositions.

ON ACYCLIC DATABASE DECOMPOSITIONS

Catriel Beeri†

Aiken Computation Laboratory

Harvard University

Cambridge, MA 02138

Moshe Y. Vardi‡

Department of Computer Science

Stanford University

Stanford, CA 94305

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Abstract

Given a universal relation scheme, presented as a set of attributes and a set of dependencies, it may be advantageous to *decompose* it into a collection of schemes, each with its own sets of attributes and dependencies, which has some desired properties. A basic requirement for such a decomposition to be useful is that the corresponding decomposition map on universal relations be *injective*. A central problem in database theory is to find the *reconstruction* map, i.e., the inverse map of an injective decomposition map. We prove here that when the decomposition, viewed as a hypergraph, is *acyclic* and the given dependencies are *full implicational dependencies*, then the reconstruction map is the *natural join*. Based on this, we show that there is a polynomial time algorithm to test for injectiveness of decompositions.

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1. Introduction

A significant portion of research on relational database theory has been concerned with the properties of *database decompositions*. The generic problem can be described as follows. Given a "universal" scheme presented as a set of attributes and a set of dependencies, what are the conditions under which it can be decomposed into a collection of schemes, each with its own sets of attributes and dependencies, having some desired properties. The properties considered were, at first, various *normal forms*. (see [Ma, Ul]). It was then realized, however, that there is a more fundamental property, called *faithfulness*, that has to be satisfied by decompositions [R1,BR,MMSU]. Intuitively, a decomposition is *faithful* if the relations corresponding to the different schemes can be updated separately.

A basic assumption underlying these ideas is that when a universal scheme is decomposed into smaller schemes, each of the universal relations associated with it is decomposed into smaller relations using the *projection* operation, i.e., each such relation is projected onto each one of the smaller schemes. For a decomposition to be faithful, we must not lose any information by decomposing the universal relations, that is the decomposition map must be *injective*. In other words, it should be possible to reconstruct the universal relations from their projections. The desirability of injectiveness is called in [BBG] the *representation principle*.

The question raised now is as follows: Given that a decomposition is faithful, what is the resulting reconstruction map? The most natural choice for this map is the *(natural) join* operation. The problem is whether indeed the reconstruction map is the join. This problem was first presented in [R1], where it is answered affirmatively for the case that only functional dependencies are given and the decomposition is into two schemes only. This result was generalized in [BR, MMSU], where the restriction on the number of schemes was removed. It was generalized further in [V2] to the case where *full implicational dependencies* are given. In contrast, it is shown there that if we allow arbitrary first-order sentences instead of full implicational dependencies, then the reconstruction map does not have to be the join.

The result in [V2] assumes that both finite and infinite databases are considered. It is more realistic, however, to assume that database are inherently finite. This is the case that we consider in this paper. While we cannot show that the reconstruction map is the join, we prove it for the important case that the decomposition is *acyclic* [FMU, BFMY]. Furthermore, in this case the condition of injectiveness is equivalent to the condition that the reconstruction map be the join. (The same result was shown independently in [CP] for the less general case where functional and join dependencies are given and and the decomposition is into two schemes.) In contrast, it is shown in [V2] that for cyclic decomposition injectiveness does not imply that the reconstruction map is the join. Finally, based on this characterization of injectiveness, we show that there is a polynomial time algorithm to test for injectiveness of decompositions.

2. Basic Definitions

2.1. Relation and Dependencies

We assume familiarity with the terminology and concepts of relational database theory as presented in [Ma, Ul]. We use $I[X]$ to denote the *projection* of the relation I on the attribute set X, and $*_j I_j$ to denote the *join* of the set $\{I_j\}$ of relations. We assume that the relations we are dealing with and, accordingly, the dependencies that refer to them are *typed*, that is, distinct attributes have disjoint domains. We also assume that all relations are finite. The *universal relation scheme* is denoted U, and a *database scheme* is a set $\mathbf{R}=\{R_1, \ldots, R_k\}$ of distinct relation scheme such that $\bigcup_{i=1}^{k} R_i = U$.

The dependencies we use here are the *full implicational dependencies* (fid's) [BV, F2], i.e., *equality-generating dependencies* (egd's) and *full tuple-generating dependencies* (ftgd's). We denote the class of relations that satisfy a set Σ of dependencies by $SAT(\Sigma)$, and we denote logical implication by $\models$.

The class of *join dependencies* [ABU,R2] is a subclass of the class of ftgd's. They were originally introduced using the following notation. A join dependency (jd) is a statement $*[\mathbf{R}]$, where $\mathbf{R}$ is a database scheme $\{R_1, \ldots, R_k\}$. It is satisfied by a relation I on U if $I = \overset{k}{\underset{i=1}{*}} I[R_i]$. Let $\mathbf{R}$ and $\mathbf{S}$ be database schemes. We say that $\mathbf{S}$ *covers* $\mathbf{R}$ if for all R in $\mathbf{R}$ there is some S in $\mathbf{S}$ such that $R \subseteq S$. It is known [BMSU] that if $\mathbf{S}$ covers $\mathbf{R}$ then $*[\mathbf{R}] \models *[\mathbf{S}]$. A *multivalued dependency* (mvd) [F1, Za] is essentially a "binary" jd, i.e., it is a jd $*[R_1, R_2]$. (Note that $R_1 \cup R_2 = U$). This notation differs from the standard arrow notations for mvd's.

Let $\mathbf{R} = \{R_1, \ldots, R_k\}$ be a database scheme. If $\mathbf{R}_1$ and $\mathbf{R}_2$ are subsets of $\mathbf{R}$, then $\mathbf{R}_1, \mathbf{R}_2$ is a partition of $\mathbf{R}$ if $\mathbf{R}_1 \cap \mathbf{R}_2 = \emptyset$ and $\mathbf{R} = \mathbf{R}_1 \cup \mathbf{R}_2$. A database scheme $\mathbf{R}$ is said to be *acyclic* [FMU,BFMY] if the jd $*[\mathbf{R}]$ is logically equivalent to the set of mvd's

$$\{*[\bigcup \mathbf{R}_1, \bigcup \mathbf{R}_2] : \mathbf{R}_1, \mathbf{R}_2 \text{ is a partition of } \mathbf{R}\}.$$

Note that one direction of the equivalence is true for any database scheme.

We will use here two properties of fid's that are shown in [V1,V3].

(1) Let Σ be a set of fid's and let τ be an mvd. If $\Sigma \not\models \tau$ then there is a relation I such that $|I| = 2$, I satisfies Σ, and I does not satisfy τ.

(2) There is a quadratic time algorithm to test for a given finite set Σ of fid's and an mvd τ whether $\Sigma \models \tau$.

2.2. Decompositions

A database on the database $\mathbf{R} = \{R_1, \ldots, R_k\}$ is a collection $\{I_1, \ldots, I_k\}$ of relations on $R_1, \ldots, R_k$, correspondingly. With each database scheme $\mathbf{R}$ we associate a *decomposition map* $\Delta_{\mathbf{R}}$. Given a relation I on U, $\Delta_{\mathbf{R}}$ applied to I yields a database on $\mathbf{R}$:

$$\Delta_{\mathbf{R}}(I) = \{I[R_1], \ldots, I[R_k]\}.$$

A decomposition map $\Delta_{\mathbf{R}}$ is *injective* with respect to a set Σ of dependencies if $\Delta_{\mathbf{R}}$ is injective on $SAT(\Sigma)$. That is, if I and J are two distinct relations in $SAT(\Sigma)$, then $\Delta_{\mathbf{R}}(I) \neq \Delta_{\mathbf{R}}(J)$. If

Δ_R is injective with respect to Σ then it has an inverse map

$$\rho_R:\{\Delta_R(I): I \in SAT(\Sigma)\} \rightarrow SAT(\Sigma),$$

such that $\rho_R(\Delta_R(I))=I$ for all I in $SAT(\Sigma)$. ρ_R is called the *reconstruction map.* If $\rho_R(I_1, \ldots, I_k)=\overset{k}{\underset{j=1}{*}} I_j$ then we say that the reconstruction map is the join. Another map associated with a database scheme R is the *project-join map* m_R defined by

$$m_R(I)=\overset{k}{\underset{j=1}{*}} I[R_j].$$

Note that a relation I satisfies *[R] exactly when $m_R(I)=I$.

3. The Main Result

Let I be a relation on U. A *permutation* on I is an injective map α from the set of values in I into itself such that the set of values for each attribute is mapped into itself. A permutation on I is essentially a vector of permutations, one for each attribute of U. We denote by $\alpha(I)$ the relation obtained by replacing simultaneously each value in I by its image under α.

Lemma 1. [BV] Let I be a relation on U and let α be a permutation on I. Then I and $\alpha(I)$ satisfy exactly the same fid's. ■

Obviously, I and $\alpha(I)$ have the same projection on each singleton attribute set $\{A\}$. They need not have, however, the same projections on larger attribute sets. A permutation α on a relation I *preserves* a database scheme R if $\Delta_R(I)=\Delta_R(\alpha(I))$.

Lemma 2. Let $R=\{R_1,R_2\}$ be a database scheme, and let $I=\{w_1,w_2\}$ be a relation on U. For every tuple w in $m_R(I)$ there exists a permutation α on I that preserves R such that w is in $\alpha(I)$.

Proof. If w is in I then take α to be the identity permutation, so we can assume that w is not in I. If however either $R_1 \subseteq R_2$ or $R_2 \subseteq R_1$, then $m_R(I)=I$, so assume that the two sets are incomparable. We can also assume that $w_1[R_1] \neq w_2[R_1]$ and $w_1[R_2] \neq w_2[R_2]$, otherwise

$m_R(I)=I$. Finally, $w_1[R_1 \cap R_2]=w_2[R_1 \cap R_2]$, since otherwise $m_R(I)=I$.

Let $w[R_1]=w_1[R_1]$ and $w[R_2]=w_2[R_2]$. Define α to be the identity on each attribute in R_1. For an attribute in R_2, α exchanges the values $w_1[A]$ and $w_2[A]$. α is well defined, since $w_1[R_1 \cap R_2]=w_2[R_1 \cap R_2]$. Now we have that $\alpha(w_1)[R_1]=w_1[R_1]$ and $\alpha(w_1)[R_2]=w_2[R_2]$, so $w=\alpha(w_1) \in \alpha(I)$. It is also easy to see that α preserves R. ■

The relationship between covering and preservation is pointed to in the following easy lemma.

Lemma 3. Let R and S be database schemes such that S covers R. If α is a permutation on a relation I that preserves S, then it also preserves R. ■

We can now prove our main result.

Theorem 1. Let Σ be a set of fid's, and let R be an acyclic database scheme. The following three conditions are equivalent:

(1) Δ_R is injective with respect to Σ.

(2) $\Sigma \models^*[R]$.

(3) ρ_R is the join.

Proof.

(1 -> 2): Suppose that Δ_R is injective with respect to Σ but $\Sigma \not\models^*[R]$. Since R is acyclic, there is a partition R_1, R_2 of R such that $\Sigma \not\models^*[S]$, where $S=\{\bigcup R_1, \bigcup R_2\}$. By property (1) of fid's, there is a relation $I=\{w_1, w_2\}$ such that I satisfies Σ but I does not satisfy $^*[S]$. Since I does not satisfy S, there is a tuple w that is in $m_S(I)$ but not in I. By Lemma 2, there is a permutation α on I such that w is in $\alpha(I)$, and α preserves S. But then we must have $I \neq \alpha(I)$, since w is not in I, and by Lemma 1, I and $\alpha(I)$ both satisfy Σ. We also have that α preserves R, because S covers R. Therefore, $\Delta_R(I)=\Delta_R(\alpha(I))$ - in contradiction to the injectiveness of Δ_R with respect to Σ.

(2 -> 3): Let I be a relation in $SAT(\Sigma)$. Since $\Sigma \models^*[R]$, $I=m_R(I)$. That is,

$$I = \overset{k}{\underset{i=1}{*}} I[R_i] = \rho_R(\Delta_R(I)),$$

where ρ_R is the join.

(3 -> 1): If ρ_R exists, then Δ_R must be injective with respect to Σ. ■

We now show that the condition of Theorem 1 can be tested efficiently.

Theorem 2. There is cubic time algorithm to test for a given finite set Σ of fid's and an acyclic database scheme R whether Δ_R is injective with respect to Σ.

Proof. By Theorem 1 it suffices to test whether $\Sigma \models *[R]$. Our strategy is to construct first a set Γ of mvd's that is logically equivalent to $*[R]$ (by acyclicity), and then to test if $\Sigma \models \tau$ for each τ in Γ. By property (2) of fid's,, each of the latter tests can be done in quadratic time. The crux of the proof is showing how to construct Γ such that $|\Gamma| < |U|$ and the construction can be done in quadratic time.

If R is acyclic then $|R| \leq U$ and there is a set Γ of mvd's that is logically equivalent to $*[R]$ such that $|\Gamma| < |R|$ [BFMY]. In order to construct Γ we have to construct first a *join forest* for R.[1] This join forest can be constructed in time linear in the size of R [TY]. Then every mvd in Γ can be constructed from the join forest in time linear in the size of R [BFMY]. Since $|\Gamma| < |U|$, the claim follows. ■

4. Concluding Remarks

We have shown that the reconstruction map is the join for quite a general situation, namely when the dependencies are fid's and the decomposition is acyclic. We note that most classes of dependencies treated in the literature are special cases of fid's. An exception is the class of *inclusion dependencies* [CFP]. Our results can be generalized to deal also with inclusion dependencies [KCV]. As for the restriction that the decomposition be acyclic there are arguments to the effect that most real life situations can be captured by such decompositions.

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[1] A join forest for R is a labeled acyclic graph with the elements of R as nodes, an edge connecting R_i to R_j is

We have also shown how to test efficiently for injectiveness. We mentioned that there is another desirable property of decompositions, called *surjectiveness* [V2]. Faithful decompositions are both injective and surjective. When only functional dependencies are given there is a polynomial time test for faithful [BH, BR, MMSU]. We do not know of any effective test when fid's are given, even when the decomposition is acyclic.

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labeled by $R_i \cap R_j$, and for each attribute A the subgraph of nodes and edges that contain A is connected.

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